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what's the point of... DIFFERENTIATION?

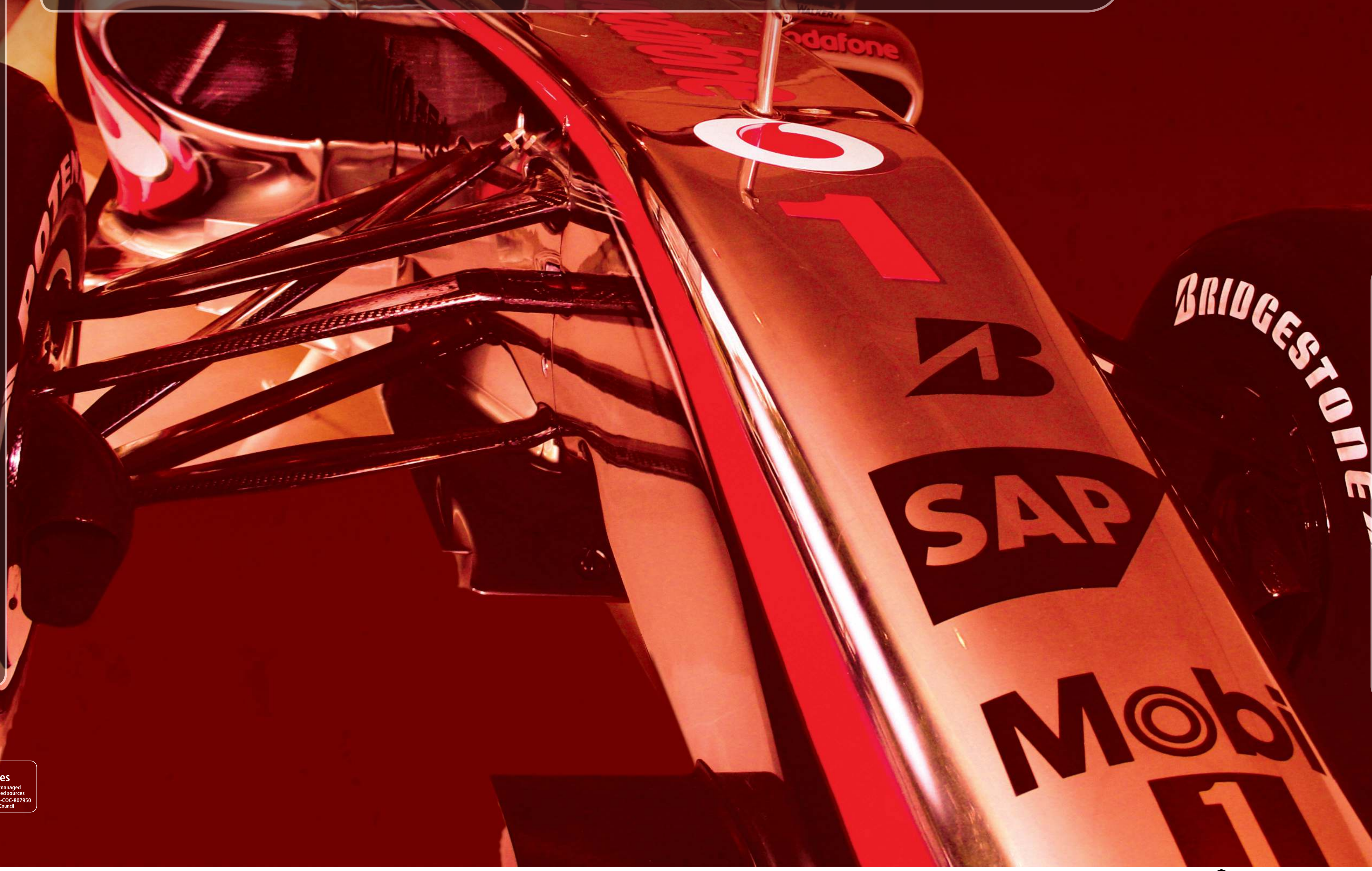
Go with the flow

Racing cars need to be able to cut through the air at high speeds and buildings need to remain standing even in very high winds. In both situations, engineers need to understand the behaviour of air as it flows around objects.

In fact, air in these situations can be thought of as a fluid and the maths behind how fluids behave is known as fluid mechanics. This covers things as diverse as how blood flows through a heart and how peanut butter moves through pipes in a food factory.

The foundation of fluid mechanics is a collection of differential equations that were derived by Claude-Louis Navier in 1822 and then developed by George Stokes in 1845. This set of differential equations are called the Navier–Stokes equations and they allow us to understand the flow of fluids.

However, even though these equations can be solved to help us understand the flow of anything that can be considered a fluid (and this includes not only gases and liquids but even things like traffic in a city and stars moving in a galaxy) they are not mathematically complete! Mathematicians have not managed to work out if these differential equations will always produce an answer and if any answer they give will always make sense. The Clay Mathematics Institute have decided that this is one of the most important unsolved problems in maths and so they have promised to give a million US dollars to the first person to fix the problem. Could you be the person who gets a million dollars for completing our understanding of the Navier–Stokes–YourName differential equations?



The need for speed

Conventional methods for detecting speeding include radar guns and roadside cameras. Increasingly, however, average-speed cameras are being introduced to Britain's roads and, with the infallible logic of maths, they are hard to deceive.

Take the example of a driver who has spotted a roadside camera. Obviously if she is speeding and spots a camera, she will slow down using the brakes as she approaches the camera. Once she has passed it, however, she will probably speed up again. If she sees another camera later in the journey, she will apply the brakes again, then speed up again once she's past it.

Now take the scenario where two average-speed cameras are placed at a known distance apart and the driver continues to use the same strategy as before. Using simple distance, velocity and acceleration equations and something called the mean value theorem it is possible to work out whether the driver has broken the speed limit, even if she is obeying the limit as she approaches the cameras.

The position function (or distance) of a travelling car can be mapped as

$$x(t) = \frac{1}{2}at^2 + v_0 t + x_0$$

(where a is acceleration, v_0 is initial velocity, x_0 is initial position and t is time). $x'(t)$ is the first derivative of the equation above with respect to t and this is a measurement of velocity (or speed).

The mean value theorem states that

$$x'(t) = \frac{x(t) - x(t_0)}{t - t_0}$$

(the rate of change in distance over the change in time – in other words the average speed).

In practice, this means that the first camera makes a note of the initial position and time ($x(t_0)$ and t_0 respectively). The second camera marks the finish position and time. By substituting the values into the equation above, it is relatively simple to work out if the driver has been speeding or not.

This method of recording average speed is very difficult to argue against particularly if you want to take it to court. The moral of the story? Drive safely!

Calculus for a healthy heart

When looking for signs of a healthy body, or otherwise, two of the things doctors measure include blood pressure and pulse. The flow of blood around the body is often referred to as haemodynamics.

The flow of blood (and rate of change of flow – differentiation!) through the body is critical to our survival. Often increase of blood pressure and/or restricted blood flow through the arteries (atherosclerosis) due to the deposit of materials such as cholesterol is a major sign that something like a heart attack could strike pretty soon.

The way to measure and monitor such phenomena are based on medical knowledge coupled with mathematical expertise including an understanding of fluid dynamics. Using Newton's laws of mass and momentum, the Navier–Stokes equations, Bernoulli equations, among others, it is possible to model the optimal blood flow through a body.

If the signs are caught early enough you may want to thank maths and medicine, amongst others, for helping to recognise the danger.

• For further information, articles and resources visit: www.moremathsgrads.org.uk • www.mathscareers.org.uk • www.cs4fn.org plus.maths.org • nrich.maths.org

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